\documentclass{article}

\usepackage[utf8]{inputenc}

\usepackage{graphicx}

\usepackage{calligra}

\DeclareMathAlphabet{\mathcalligra}{T1}{calligra}{m}{n}

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\newcommand{\scripty}[1]{\ensuremath{\mathcalligra{#1}}}

\begin{document}

\begin{titlepage}

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\textsc{\huge Examining Radiation of a Four Dipole System}\newline

\includegraphics{Suny Logo.png}

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{\large \bfseries SUNY Polytechnic Institute}\\

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\textsc{\large Electrical Engineering/Physics}

\newline\large Spring 2018\newline

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\textsc{\LARGE \bf Abstract}\\\\

This paper studies a system of four eclectic dipoles and the radiation that the system produces. The radiation patern for this system will be found by first analyzing the scalar and vector potentials caused by the dipoles and furthermore examine the electric and magnetic fields that the dipoles produce. The Poynting vector and intensity profile for the system will also be examined. The system contains two dipoles of opposite charge on the vertical z-axis and two oppositely charged dipoles on the horizontal y-axis which has a very symmetric layout. In order to build up the techniques needed to examine the four dipole system, this paper will analyze the radiation produced by a single dipole and also the radiation produced by a system of two oppositely charged dipoles on the vertical z-axis. The radiation plots of each system will be compared and analyzed as well.

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\section{Introduction}

An electric dipole is a separation of charges of the opposite sign with the same absolute value separated by a very small distance. When these charges accelerate it creates changing currents which can produce electromagnetic waves. The energy created by these fields that continue to infinity is called radiation. As we know, electromagnetic waves propagate from their origin out to infinity. In this paper, we will be examining a point that will be effected by the radiation of four electric dipoles. Because we are dealing with a point a significant distance away from the electric dipoles we will have to consider retarded times and further more the retarded potentials and fields.

(Explain more stuff here...)

\subsection{Single Dipole Radiation}

Before we can examine a system of four electric dipoles, we must understand how a single electric dipole radiates. Consider the figure below [Figure ...].

[INSERT figure HERE]

The figure shows that there are two charges that are separated by a distance of d and we can imagine that they are connected by a very small and thin wire in which the the charge moves back and forth at and angular frequency \(\omega\). We can say that the charge is

\begin{equation}

q(t) = q\_0\cos(\omega t)

\end{equation}

which when oscillating will yield an electric dipole

\begin{equation}

\vec{p}(t) = p\_0\cos(\omega t)

\end{equation}

where \(p\_0 = q\_0 d\) is the maximum value of the dipole moment. We can now use the generalized equations for retarded potentials which can be seen below

\begin{equation}

V(\vec{r},t) = \frac{1}{4\pi\epsilon\_0}\int\_{a}^{b}<\frac{\rho(\vec{r'}, t\_r)}{\scripty{r}}>d\tau'

\end{equation}

and

\begin{equation}

\vec{A}(\vec{r},t) = \frac{\mu\_0}{4\pi}\int\_{a}^{b}<\frac{\vec{J}(\vec{r'}, t\_r)}{\scripty{r}}>d\tau'

\end{equation}

Plugging into the equation for retarded potential we get the following

\begin{equation}

V(\vec{r},t) = \frac{1}{4\pi\epsilon\_0}{\frac{q\_0\cos[\omega(t-\scripty{r}\_+/c)]}{\scripty{r}\_+}}-\frac{q\_0\cos[\omega(t-\scripty{r}\_-/c)]}{\scripty{r}\_-}

\end{equation}

We can now apply the law of cosines to get the equations

\begin{equation}

\scripty{r}\_\pm = \sqrt{r^2\mp rd\cos\theta+(d/2)^2}

\end{equation}

To make this dipole into a perfect dipole, the distance between the two charges must be very small which means we will make the approximation that \(d << r\). If d were to be zero there would be no potential at all so we will carry an expansion to the first order in d. Thus we can say

\begin{equation}

\scripty{r}\_\pm = r(1\mp \frac{d}{2r}\cos\theta)

\end{equation}

and therefore

\begin{equation}

\frac{1}{\scripty{r}\_\pm} = \frac{1}{r}(1\pm \frac{d}{2r}\cos\theta)

\end{equation}

We will now plug in the script r into \(\cos[\omega(t-\frac{\scripty{r}\pm}{c})]\) which will yield the following:

\begin{equation}

\cos[\omega(t-\frac{\scripty{r}\pm}{c})] = \cos[\omega(t-\frac{r}{c}\pm \frac{\omega d}{2c}\cos\theta]

\end{equation}

\begin{equation}

= \cos[\omega(t-\frac{r}{c})]\cos(\frac{\omega d}{2c}\cos\theta)\mp \sin[\omega(t-\frac{r}{c})]\sin\theta.

\end{equation}

In the perfect dipole limit we can further make the approximation that \(d << \frac{c}{\omega}\) which allows us to further simplify equation 10 as

\begin{equation}

\cos[\omega(t-\frac{\scripty{r}\_\pm}{c})] = \cos[\omega(t-\frac{r}{c})]\mp \frac{\omega d}{2c}\cos\theta\sin[\omega(t-\frac{r}{c})].

\end{equation}

Now we can plug equations 8 and 11 into equation 5 and get the following

\begin{equation}

V(r,\theta,t) = \frac{p\_0 \cos\theta}{4\pi\epsilon\_0r}{-\frac{\omega}{c}\sin[\omega(t-\frac{r}{c}]+\frac{1}{r}\cos[\omega(t-\frac{r}{c}]}

\end{equation}

Since we only are interested in the fields that survive at long distances in the radiation zone we can make the approximation that \(r >> \frac{c}{\omega}\) which means \(r >> \lambda\). In this region the potential in equation 12 will reduce to

\begin{equation}

V(r,\theta,t)= \frac{-p\_0\omega}{4\pi\epsilon\_0 c}(\frac{\cos\theta}{r})\sin[\omega(t-\frac{r}{c})].

\end{equation}

The vector potential for this dipole can be found by determining the current flowing through the wire which would give

\begin{equation}

\vec{I}(t) = \frac{dq}{dt}\hat{z} = -q\_0\omega\sin(\omega t)\hat{z}

\end{equation}

which plugging into equation 4 will give

\begin{equation}

\vec{A}(\vec{r},t) = \frac{\mu\_0}{4\pi}\int\_{-\frac{d}{2}}^{\frac{d}{2}}\frac{-q\_0\omega\sin[\omega(t-\frac{\scripty{r}}{c})])\hat{z}}{\scripty{r}}dz

\end{equation}

Since the integration produces a factor of d, we can replace the integrand by its value at the center to the first order which will give

\begin{equation}

\vec{A}(r,\theta,t) = -\frac{\mu\_0p\_0\omega}{4\pi r}\sin[\omega(t-\frac{r}{c}]\hat{z}

\end{equation}

Now that we have acquired the vector and scalar potentials, we can compute the electric and magnetic fields produced by the dipole. We will use the two equations below:

\begin{equation}

\vec{E}=-\bigtriangledown V-\frac{\partial{\vec{A}}}{\partial{t}}

\end{equation}

\begin{equation}

\vec{B}=\vec{\bigtriangledown}\times\vec{A}

\end{equation}

Starting with eclectic field we will solve for the gradient of the scalar potential.

\begin{equation}

\bigtriangledown V = \frac{-p\_0\omega}{4\pi\epsilon\_0c}{\cos\theta(\frac{-1}{r^2}\sin[\omega(t-\frac{r}{c})]-\frac{\omega}{rc}\cos[\omega(t-\frac{r}{c})])\hat{r}-\frac{\sin\theta}{r^2}\sin[\omega(t-\frac{r}{c})]}

\end{equation}

\begin{equation}

\bigtriangledown V = \frac{p\_0\omega^2}{4\pi\epsilon\_0c^2}(\frac{\cos\theta}{r})\cos[\omega(t-\frac{r}{c})]\hat{r}.

\end{equation}

Now for the partial derivative for the vector potential with respect to t.

\begin{equation}

\frac{\partial{\vec{A}}}{\partial{t}} = -\frac{\mu\_0p\_0\omega^2}{4\pi r}\cos[\omega(t-\frac{r}{c}](\cos\theta\hat{r}-\sin\theta\hat{\theta})

\end{equation}

Now we will take the curl of the vector potential.

\begin{equation}

\vec{B}=\vec{\bigtriangledown}\times\vec{A} = -\frac{\mu\_0p\_0\omega}{4\pi r}{\frac{\omega}{c}\sin\theta\cos[\omega(t-\frac{r}{c})]+\frac{\sin\theta}{r}\sin[\omega(t-\frac{r}{c})]}\hat{\phi}.

\end{equation}

Putting together the equations we just solved for and dropping terms to satisfy our third approximation, we can finally say

\begin{equation}

\vec{E} = -\bigtriangledown V - \frac{\partial{\vec{A}}}{\partial{t}} = -\frac{\mu\_0p\_0\omega^2}{4\pi}(\frac{\sin\theta}{r})\cos[\omega(t-\frac{r}{c})]\hat{\theta}.

\end{equation}

\begin{equation}

\vec{B}=\vec{\bigtriangledown}\times\vec{A} = -\frac{\mu\_0p\_0\omega^2}{4\pi c}(\frac{\sin\theta}{r})\cos[\omega(t-\frac{r}{c})]\hat{\phi}.

\end{equation}

Now that we have the electric and magnetic fields, we can find the energy radiated by an oscillating electric dipole which will be determined by solving for the pointing vector.

\begin{equation}

\vec{S} = \frac{1}{\mu\_0}(\vec{E}\times\vec{B}) = \frac{\mu\_0}{c}{\frac{p\_0\omega^2}{4\pi}(\frac{\sin\theta}{r})\cos[\omega(t-\frac{r}{c})]}^2\hat{r}.

\end{equation}

The intensity of the dipole can by obtained by averaging over a complete cycle in time.

\begin{equation}

<\vec{S}> = (\frac{\mu\_0p^2\_0\omega^4}{32\pi^2c})\frac{\sin^2\theta}{r^2}\hat{r}.

\end{equation}

And finally the total power radiated is found by integrating \(<\vec{S}>\) over a sphere of radius r.

\begin{equation}

<P> = \int<\vec{S}>\cdot d\vec{a} = \frac{\mu\_0p^2\_0\omega^4}{32\pi^2c}\int\frac{\sin^2\theta}{r^2}r^2\sin\theta d\theta d\phi = \frac{\mu\_0p^2\_0\omega^4}{12\pi c}.

\end{equation}

When we plot the intensity profile we get the following image:

[INSET IMAGE HERE]

One can see that the intensity profile looks like the shape of a donut. We will come back to this image for comparison when examining the intensity profile of more complex systems involved multiple electric dipoles.

\subsection{Two Oppositely Charged Electric Dipoles}

Now that we have looked at the radiation pattern for a single electric dipole, we will observe a system of two electric dipoles so that we can understand how symmetry will effect the radiation. When solving this system we will still use the same approximations as before which were:

\textbf{Approximation 1:}

\begin{equation}

d << r

\end{equation}

\textbf{Approximation 2:}

\begin{equation}

d << c/\omega

\end{equation}

\textbf{Approximation 3:}

\begin{equation}

r >> c/\omega

\end{equation}

We will use these approximations and what we have learned from finding the radiation of a single electric dipole to solve for a system of two electric dipoles along the same vertical axis. A figure of the system can be seen below:

\begin{figure}

\includegraphics[width=\textwidth]{single-dipole-system.JPG}

\caption{System of four electric dipoles}

\label{fig:boat1}

\end{figure}

We will use many of the same equations that were used in section 3 with a few small changes. First let us solve for the potentials. In this system the potentials are now affected by two different dipoles so we will find the potentials of each individually and add them together to get our total potentials. The first step in solving for the potentials is establishing our distances and angles. We will start by doing the following.

\begin{equation}

\scripty{r}^2\_\pm = r^2 + (\frac{d}{2})^2 \mp 2r(\frac{d}{2})\cos\theta

\end{equation}

\begin{equation}

\scripty{r}\_\pm = \sqrt{r^2 + (\frac{d}{2})^2 \mp 2r(\frac{d}{2})\cos\theta}

\end{equation}

\begin{equation}

\scripty{r}\_\pm = r\sqrt{1 \mp (\frac{d}{r}\cos\theta)} = r(1 \mp \frac{d}{2r}\cos\theta)

\end{equation}

Next we will solve for \(\cos\theta\_\pm\).

\begin{equation}

\cos\theta\_\pm = \frac{r\cos\theta\mp\frac{d}{2}}{r\_\pm} = r(\cos\theta\mp\frac{d}{2r})\frac{1}{r}(1\pm\frac{d}{2r}\cos\theta)

\end{equation}

Simplifying the last equation we get

\begin{equation}

\cos\theta\_\pm = \cos\theta\pm\frac{d}{2r}\cos^2\theta\mp\frac{d}{2r} = \cos\theta\mp\frac{d}{2r}(1-\cos^2\theta)

\end{equation}

Which allows us to finally say

\begin{equation}

\cos\theta\_\pm = \cos\theta \mp \frac{d}{2r}\sin^2\theta.

\end{equation}

Lastly, before plugging into our potential equation we will solve for \(\sin[\omega(t-\frac{\scripty{r}\_\pm}{c})]\).

\begin{equation}

\sin[\omega(t-\frac{\scripty{r}\_\pm}{c})] = \sin[\omega(t-\frac{r}{c}(1\mp\frac{d}{2r}\cos\theta))] = \sin(\omega t\_0 \pm \frac{\omega d}{2c}\cos\theta)

\end{equation}

where \(t\_0 = t-\frac{r}{c}\). Further simplifying this equation we get

\begin{equation}

\sin(\omega t\_0 \pm \frac{\omega d}{2c}\cos\theta) = \sin(\omega t\_0)\cos(\frac{\omega d}{2c}\cos\theta)\pm\cos(\omega t\_0)\sin(\frac{\omega d}{2c}\cos\theta)

\end{equation}

which can be finally written as

\begin{equation}

\sin[\omega(t-\frac{\scripty{r}\_\pm}{c})] = \sin(\omega t\_0)\pm \frac{\omega d}{2c}\cos\theta\cos(\omega t\_0)

\end{equation}

Now we have everything we need to plug into our potential equations. First we will solve for the scalar potential using the following

\begin{equation}

V = \frac{-p\_0\omega}{4\pi\epsilon\_0c}(\frac{\cos\theta}{r})\sin[\omega(t-\frac{r}{c})]

\end{equation}

\begin{equation}

V\_{\pm} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(1\pm\frac{d}{2r}\cos\theta)(\cos\theta\mp\frac{d}{2r}\sin^2\theta)(\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0))]

\end{equation}

\begin{equation}

V\_{\pm} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(\cos\theta\mp\frac{d}{2r}\sin^2\theta+\_-\frac{d}{2r}\cos^2\theta)(\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0))]

\end{equation}

\begin{equation}

V\_{\pm} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[\cos\theta\sin(

\omega t\_0) \pm \frac{\omega d}{2c}\cos^2\theta\cos(\omega t\_0) \pm \frac{d}{2r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

\newline\newline

We will call the sum of these two expressions V.Adding together the two expressions we get

\begin{equation}

V = \frac{-p\_0\omega}{4\pi\epsilon\_0cr}[\frac{\omega d}{c}\cos^2\theta\cos(\omega t\_0)+\frac{d}{r}(\cos^2\theta-\sin^2\theta)\sin\omega t\_0)]

\end{equation}

\begin{equation}

V = \frac{-p\_0\omega^2 d}{4\pi\epsilon\_0c^2r}[\cos^2\theta\cos(\omega t\_0)+\frac{c}{\omega r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

And using our third approximation which said \(r >> \omega/c\) (equation 3), we can write V as

\begin{equation}

V = \frac{-p\_0\omega^2 d}{4\pi\epsilon\_0c^2r}[\cos^2\theta\cos(\omega t\_0)]

\end{equation}

and subbing back in \(t\_0 = t-\frac{r}{c}\) we get

\begin{equation}

V = \frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2r}\cos^2\theta\cos[\omega(t - \frac{r}{c})]

\end{equation}

Next we will solve for our vector potential by plugging into the following

\begin{equation}

\vec{A} = \frac{-\mu\_0p\_0\omega}{4\pi r}\sin[\omega(t-\frac{r}{c})]]

\end{equation}

\begin{equation}

\vec{A\_{\pm}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r\_{\pm}}\sin[\omega(t-\frac{r\_{\pm}}{c})]

\end{equation}

\begin{equation}

\vec{A\_{\pm}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r}[(1\pm\frac{d}{2r}\cos\theta)[\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0)]]\hat{z}

\end{equation}

\begin{equation}

\vec{A\_{\pm}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r}[(\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0)\pm\frac{d}{2r}\cos\theta\sin(\omega t\_0)\hat{z}

\end{equation}

Now we can say \(\vec{A} = \vec{A}\_+ + \vec{A}\_-\).

\begin{equation}

\vec{A} = -\frac{\mu\_0p\_0\omega}{4\pi r}[\frac{\omega d}{c}\cos\theta\cos(\omega t\_0)+\frac{d}{r}\cos\theta\sin(\omega t\_0)]\hat{z}

\end{equation}

Then factoring out \(\frac{\omega d\cos\theta}{c}\) we are get

\begin{equation}

\vec{A} = - \frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta[\cos(\omega t\_0)+\frac{c}{\omega r}\sin(\omega t\_0)]]\hat{z}.

\end{equation}

And again in our radiation zone we can apply our third assumption (equation 3) which will eliminate the second term giving us

\begin{equation}

\vec{A} = -\frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta\cos(\omega t\_0)\hat{z}.

\end{equation}

and subbing back in for \(t\_0\)

\begin{equation}

\vec{A} = -\frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta\cos[\omega(t-\frac{r}{c})]\hat{z}.

\end{equation}

As before, we will use these potentials to solve for the electric and magnetic fields produced by the dipoles. We will use the same equations as before which are again shown below

\begin{equation}

\vec{E}=-\bigtriangledown V-\frac{\partial{\vec{A}}}{\partial{t}}

\end{equation}

\begin{equation}

\vec{B}=\vec{\bigtriangledown}\times\vec{A}

\end{equation}

We will start by first solving for the gradient of the scalar potential.

\begin{equation}

\bigtriangledown V = \frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2}\cos^2\theta(\frac{\omega r \sin[[\omega(t-\frac{r}{c})]-\cos[\omega(t-\frac{r}{c})]}{r^2})\hat{r} + \frac{p\_0\omega^2d}{2\pi\epsilon\_0c^2r^2}\cos(\theta)\sin(\theta)\cos[\omega(t-\frac{r}{c})]\hat{\theta}

\end{equation}

And using the approximation that \(r >> \frac{c}{\omega}\) we can say

\begin{equation}

\bigtriangledown V = \frac{-p\_0\omega^3d}{4\pi\epsilon\_0c^3}\frac{\cos^2(\theta)}{r}\sin[\omega(t-\frac{r}{c})]\hat{r}

\end{equation}

Lastly, we will solve for the partial derivative of the vector potential with respect to time.

\begin{equation}

\frac{\partial{\vec{A}}}{\partial{t}} = \frac{\mu\_0p\_0\omega^3}{4\pi rc}\cos\theta\sin[\omega(t-\frac{r}{c})](\cos\theta\hat{r}-\sin\theta\hat{\theta})

\end{equation}

Finally, we can plug into our field equations giving us:

\begin{equation}

\vec{E} = \frac{\mu\_0p\_0\omega^3d}{4\pi cr}\sin\theta\cos\theta\sin[\omega(t-\frac{r}{c})]\hat{\theta}

\end{equation}

\begin{equation}

\vec{B} = \frac{\mu\_0p\_0\omega^3d}{4\pi c^2r}\sin\theta\cos\theta\sin[\omega(t-\frac{r}{c})]\hat{\phi}

\end{equation}

Now that we have our field vectors, we can solve for the Poynting vector

\begin{equation}

\vec{S} = \frac{1}{\mu\_0}(\vec{E}\times\vec{B})

\end{equation}

\begin{equation}

\vec{S} = \frac{1}{\mu\_0}[\frac{-\mu\_0p\_0\omega^3d}{4\pi rc}\sin\theta\cos\theta\sin[\omega(t-\frac{r}{c})]]^2\hat{r}

\end{equation}

Averaging this vector over a complete cycle in time we get the intensity to be

\begin{equation}

<\vec{S}> = \frac{1}{2\mu\_0c}[\frac{-\mu\_0p\_0\omega^3d}{4\pi rc}\sin\theta\cos\theta]^2

\end{equation}

Finally we can get the total radiated power by integrating the intensity over a sphere of radius r

\begin{equation}

P=\int\_{a}^{b}<\vec{S}>\cdot d\vec{a}

\end{equation}

\begin{equation}

P=\frac{1}{\mu\_0}[\frac{-\mu\_0p\_0\omega^3d}{4\pi c}]^2\int\sin^2\theta\cos^2\theta\sin\theta d\theta d\phi

\end{equation}

Which will finally give us

\begin{equation}

P=\frac{\mu\_0p\_0^2d^2\omega^6}{60\pi c^3}

\end{equation}

[PLOT THE INTENSITY PROFILE HERE]

\section{Electric Dipole Calculations}

In this section we will be looking at the case of four electric dipoles, each one positioned 90 degrees from the next. A figure of the electric dipoles can be seen in the figure below. For our calculations we will be using three main approximations: \linebreak

\textbf{Approximation 1:}

\begin{equation}

d << r

\end{equation}

\textbf{Approximation 2:}

\begin{equation}

d << c/\omega

\end{equation}

\textbf{Approximation 3:}

\begin{equation}

r >> c/\omega

\end{equation}

\begin{figure}[h!]

\centering

\includegraphics[width=\textwidth]{Diagram.PNG}

\caption{System of four electric dipoles}

\label{fig:dipole-addition}

\end{figure}

To make the system simpler to solve we will break it into two systems containing two symmetrical dipoles each which will allow us to use the same techniques as before and add the results at the end. The systems will looks as follows in Figure 4.

\begin{figure}[h!]

\includegraphics[width=\textwidth]{dipole-addition.JPG}

\caption{System of four electric dipoles}

\label{fig:4-dipoles}

\end{figure}

\subsection{Scalar and Vector Potentials}

To find the correct distances from the electric dipoles to point P we will use the law of cosines which is: \(a^2=b^2+c^2-2bc\cos A\). We will start off by solving for the distance to point P from the electric dipoles on the z-axis in the vertical system which is seen below in Figure 5.

\begin{figure}[h!]

\includegraphics[width=\textwidth]{single-dipole-system.JPG}

\caption{System of four electric dipoles}

\label{fig:vertical-dipoles}

\end{figure}

The dipoles are separated by a distance d from each other. First lets look at the electric dipole on the positive z-axis. We will use the equation:

\begin{equation}

r\_+^2=r^2+\frac{d}{2}^2-2r\frac{d}{2}\cos\theta

\end{equation}

\begin{equation}

r\_+ = \sqrt{r^2+\frac{d}{2}^2-2r\frac{d}{2}\cos\theta}

\end{equation}

\begin{equation}

r\_+ = \sqrt{r^2(1+\frac{d^2}{4r^2}-\frac{d}{r}\cos\theta)}

\end{equation}

And using the first assumption \(d << r \) (equation 1),

\begin{equation}

r\_+ = r\sqrt{(1-\frac{d}{r})\cos\theta)}

\end{equation}

\begin{equation}

r\_+ = r(1-\frac{d}{r}\cos\theta)^{1/2}

\end{equation}

Using: \((1+\epsilon)^n\approx(1+n\epsilon)\):

\begin{equation}

r\_+ = r(1-\frac{d}{2r}\cos\theta)

\end{equation}

Which then means that

\begin{equation}

\frac{1}{r\_+} = \frac{1}{(r(1-\frac{d}{2r}\cos\theta))}

\end{equation}

Again using: \((1+\epsilon)^n\approx(1+n\epsilon)\), gives us

\begin{equation}

\frac{1}{r\_+} = \frac{1}{r}(1+\frac{d}{2r}\cos\theta)

\end{equation}

Now looking at the electric dipole on the negative z-axis. Again we will use the law of cosines. However, in this case the angle being used is \(180-\theta\). This will flip the sign of the cos term in the original equation thus giving us:

\begin{equation}

r\_-^2=r^2+\frac{d}{2}^2+2r\frac{d}{2}\cos\theta

\end{equation}

\begin{equation}

r\_-=\sqrt{^2+\frac{d}{2}^2+2r\frac{d}{2}\cos\theta}

\end{equation}

Similarly, we will get something that looks like equation 9 except with the opposite sign, giving us

\begin{equation}

r\_- = r(1+\frac{d}{2r}\cos\theta)

\end{equation}

which will similar to above will yield

\begin{equation}

\frac{1}{r\_-} = \frac{1}{r}(1-\frac{d}{2r}\cos\theta)

\end{equation}

Now we will solve for the distances to point P from the electric dipoles an the y-axis as shown in the horizontal system in Figure 6.

\begin{figure}

\includegraphics[width=\textwidth]{horizontal-dipole.JPG}

\caption{System of four electric dipoles}

\label{fig:horizontal-dipoles}

\end{figure}

The dipoles in this system are separated by a distance l. First we will solve for the one labeled \(r\_R\). We will start off again by applying the law of cosines giving us

\begin{equation}

r\_R^2 = r^2+\frac{l}{2}^2-2r\frac{l}{2}\theta'

\end{equation}

where \(\theta'\) is equal to \(\frac{\pi}{2}-\theta\). By taking the square root of both sides we get

\begin{equation}

r\_R = \sqrt{r^2+\frac{l}{2}^2-2r\frac{l}{2}\cos\theta'}

\end{equation}

Similar to \(r\_+\) we know our result will be

\begin{equation}

r\_R = r(1-\frac{l}{2r}\cos\theta')

\end{equation}

but plugging in for \(\theta'\) we will get \(\cos(\frac{\pi}{2}-\theta)\) which we know equals \(sin(\theta)\). Therefor, \(r\_R\) will read

\begin{equation}

r\_R = r(1-\frac{1}{2r}\sin\theta)

\end{equation}

which means that

\begin{equation}

\frac{1}{r\_R} = \frac{1}{r}(1+\frac{l}{2r}\sin\theta)

\end{equation}

Now to look at \(r\_L\). Applying the law of cosines again we get

\begin{equation}

r\_L^2 = r^2+\frac{l}{2}^2-2r\frac{l}{2}\cos\theta''

\end{equation}

where \(\theta''\) is equal to \(\theta+\frac{\pi}{2}\). By taking the square root of both sides we get

\begin{equation}

r\_L = \sqrt{r^2+\frac{l}{2}^2-2r\frac{l}{2}\cos\theta}

\end{equation}

Similar to \(r\_R\) we know our result will be

\begin{equation}

r\_L = r(1-\frac{l}{2r}\cos\theta'')

\end{equation}

but plugging in for \(\theta''\) we will get \(\cos(\theta+\frac{\pi}{2})\) which we know equals \(-\sin\theta)\). Therefor, \(r\_L\) will read

\begin{equation}

r\_L = r(1+\frac{l}{2r}\sin\theta)

\end{equation}

which means that

\begin{equation}

\frac{1}{r\_L} = \frac{1}{r}(1-\frac{1}{2r}\sin\theta).

\end{equation}

In summary, we have solved for all of the distances from the electric dipoles to the point P. The four equations are restated below:

\begin{equation}

\frac{1}{r\_+} = \frac{1}{r}(1+\frac{d}{2r}\cos\theta)

\end{equation}

\begin{equation}

\frac{1}{r\_-} = \frac{1}{r}(1-\frac{d}{2r}\cos\theta)

\end{equation}

\begin{equation}

\frac{1}{r\_R} = \frac{1}{r}(1+\frac{l}{2r}\sin\theta)

\end{equation}

\begin{equation}

\frac{1}{r\_L} = \frac{1}{r}(1-\frac{l}{2r}\sin\theta)

\end{equation}

By solving for the distances in the form of \(\frac{1}{r}\) will allow us to plug them directly into the scalar and vector potential equations.

Now that we have solved for each one of these distance we want to plug into the general equation for scalar potential which is

\begin{equation}

V(r,\theta,t)=\frac{-p\_0\omega}{4\pi\epsilon\_0c}\frac{\cos\theta}{r}sin[\omega(t-\frac{r}{c})]

\end{equation}

where r is each r solved above and each \(\cos\theta\) is from the corresponding \(\theta\) values shown in Figure 1. So now let us solve for each \(\cos\theta\). First we will solve for \(\cos\theta\_+\). Lets say

\begin{equation}

\cos\theta\_+ = \frac{h\_+}{r\_+} = (r\cos\theta-\frac{d}{2})(\frac{1}{r})(1+\frac{d}{2r}\cos\theta)

\end{equation}

\begin{equation}

\cos\theta\_+ = \cos\theta+\frac{d}{2r}\cos^2\theta-\frac{d}{2r}-\frac{d^2}{4r^2}\cos\theta

\end{equation}

Remembering our first assumption where \(d<<r\) we can say

\begin{equation}

\cos\theta\_+ = \cos\theta+\frac{d}{2r}\cos^2\theta-\frac{d}{2r}

\end{equation}

\begin{equation}

\cos\theta\_+ = \cos\theta-\frac{d}{2r}(1-\cos^2\theta)

\end{equation}

which will give us

\begin{equation}

\cos\theta\_+ = \cos\theta-\frac{d}{2r}\sin^2\theta.

\end{equation}

Similarly, for \(\cos\theta\_-\) we will get

\begin{equation}

\cos\theta\_- = \cos\theta+\frac{d}{2r}\sin^2\theta.

\end{equation}

We will do a similar process for the dipoles on the y-axis. Lets look at \(\cos\theta\_R\) first.

\begin{equation}

\cos\theta\_R = \frac{h\_R}{r\_R} = (r\sin\theta-\frac{l}{2})(\frac{1}{r})(1+\frac{l}{2r}\sin\theta)

\end{equation}

\begin{equation}

\cos\theta\_R = sin\theta+\frac{l}{2r}\sin^2\theta-\frac{l}{2r}-\frac{l^2}{4r^2}\sin\theta

\end{equation}

And like before the \(\frac{l^2}{4r^2}\sin\theta\) will drop due to our assumptions leaving us with

\begin{equation}

\cos\theta\_R = \sin\theta+\frac{l}{2r}\sin^2\theta-\frac{l}{2r}

\end{equation}

\begin{equation}

\cos\theta\_R = \sin\theta-\frac{l}{2r}(1-\sin^2\theta)

\end{equation}

Finally giving us

\begin{equation}

\cos\theta\_R = \sin\theta-\frac{l}{2r}\cos^2\theta.

\end{equation}

Looking at \(\cos\theta\_L\) we have an equation similar to \(cos\theta\_R\) with the opposite sign giving us

\begin{equation}

\cos\theta\_L = \sin\theta+\frac{l}{2r}\cos^2\theta.

\end{equation}

Now we have all four equations for \(\cos\theta\) (equations 35, 36, 41, and 42). They are shown again below

\[\cos\theta\_+ = \cos\theta-\frac{d}{2r}\sin^2\theta.\]

\[\cos\theta\_- = \cos\theta+\frac{d}{2r}\sin^2\theta.\]

\[\cos\theta\_R = \sin\theta-\frac{l}{2r}\cos^2\theta.\]

\[\cos\theta\_L = \sin\theta+\frac{l}{2r}\cos^2\theta.\]

Now we are ready to plug into the general equation to get the scalar potential contribution for each electric dipole. Plugging the parts in for each dipole we are given these four equations

\begin{equation}

V\_+=\frac{-p\_0\omega}{4\pi\epsilon\_0c}(\frac{\cos\theta-\frac{d}{2r}\sin^2\theta}{r(1-\frac{d}{2r}\cos\theta)})\sin[\omega(t-\frac{r(1-\frac{d}{2r}\cos\theta)}{c})]

\end{equation}

\begin{equation}

V\_-=\frac{p\_0\omega}{4\pi\epsilon\_0c}(\frac{\cos\theta+\frac{d}{2r}\sin^2\theta}{r(1+\frac{d}{2r}\cos\theta)})\sin[\omega(t-\frac{r(1+\frac{d}{2r}\cos\theta)}{c})]

\end{equation}

\begin{equation}

V\_R=\frac{-p\_0\omega}{4\pi\epsilon\_0c}(\frac{\sin\theta-\frac{l}{2r}\cos^2\theta}{r(1-\frac{1}{2r}\sin\theta)})\sin[\omega(t-\frac{r(1-\frac{1}{2r}\sin\theta)}{c})]

\end{equation}

\begin{equation}

V\_L=\frac{p\_0\omega}{4\pi\epsilon\_0c}(\frac{\sin\theta+\frac{l}{2r}\cos^2\theta}{r(1+\frac{1}{2r}\sin\theta)})\sin[\omega(t-\frac{r(1+\frac{1}{2r}\sin\theta)}{c})]

\end{equation}

In order to get the total potential at point P, we need to add together the potentials of each dipole. However, in order to add of of these equations, it will be easier to go back to the original form seen again below (equation NUMBER) and add them together two at a time.

\[V = \frac{-p\_0\omega}{4\pi\epsilon\_0c}(\frac{\cos\theta}{r})\sin[\omega(t-\frac{r}{c})]\]

The easiest way to solve for the potentials in this format is to solve for \(\sin[\omega(t-\frac{r\_{\pm}}{c})]\) and \(\cos\theta\_{\pm}\) first. Once we have solved for these pieces, we can plug into the general equation that was shown above in equation blank. This will allow us to solve for the contribution of potential for dipoles on the x axis. We will have to repeat these calculations in order to get \(\sin[\omega(t-\frac{r\_{RL}}{c})]\) and \(\cos\theta\_{RL}\) which will allow us to solve for the contribution of potential from the dipoles on the y axis.

\begin{equation}

\sin[\omega(t-\frac{r\_{\pm}}{c})] = \sin[\omega(t-\frac{r}{c}(1\pm\frac{d}{2r}\cos\theta)]

\end{equation}

where

\begin{equation}

\sin[\omega(t-\frac{r}{c}\mp\frac{d}{2c}\cos\theta)] = \sin(\omega t\_0\mp\frac{d\omega}{2c}\cos\theta)

\end{equation}

Then we could say

\begin{equation}

\sin[\omega(t-\frac{r\_{\pm}}{c})]=\sin[\omega t\_0 \pm \frac{\omega d}{2c}\cos\theta]

\end{equation}

where \(t\_0 = t-\frac{r}{c}\).

Now using the identity:

\begin{equation}

\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)\pm\cos(\alpha)\sin(\beta)

\end{equation}

we can write the equation below as

\begin{equation}

\sin[\omega t\_0 \pm \frac{\omega d}{2c}\cos\theta] = \sin(\omega t\_0)\cos(\frac{\omega d}{2c}\cos\theta)\pm\cos(\omega t\_0)\sin[\frac{\omega d}{2c}\cos\theta]

\end{equation}

We can simplify this further by saying

\begin{equation}

\cos(\frac{\omega d}{2c}\cos\theta) = 1

\end{equation}

and

\begin{equation}

\sin[\frac{\omega d}{2c}\cos\theta] = \frac{\omega d}{2c}\cos\theta

\end{equation}

Now we can finally write

\begin{equation}

\sin[\omega t\_0 \pm \frac{\omega d}{2c}\cos\theta] = \sin(\omega t\_0) \pm \frac{\omega d}{2c}\cos\theta\cos(\omega t\_0)

\end{equation}

Not lets simplify \(\cos\theta\_{\pm}\).

\begin{equation}

\cos\theta\_{\pm} = \frac{r\cos\theta \mp \frac{d}{2}}{r\_{\pm}} = r(\cos\theta \mp \frac{d}{2r})(\frac{1}{r})(1 \pm \frac{d}{2r}\cos\theta)

\end{equation}

when multiplied together will give us

\begin{equation}

\cos\theta \pm \frac{d}{2r}\cos^2\theta \mp \frac{d}{2r} = \cos\theta \mp \frac{d}{2r}(1-\cos^2\theta) = \cos\theta \mp \frac{d}{2r}\sin^2\theta

\end{equation}

So we can finally say

\begin{equation}

\cos\theta\_{\pm} = \cos\theta \mp \frac{d}{2r}\sin^2\theta

\end{equation}

Using these identities, we can write out potentials in a form that is easier to combine. Lets start by looking at \(V\_+\) and \(V\_-\) (the potentials on the y axis). We can write them as follows:

\begin{equation}

V\_{\pm} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(1\pm\frac{d}{2r}\cos\theta)(\cos\theta\mp\frac{d}{2r}\sin^2\theta)(\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0))]

\end{equation}

\begin{equation}

V\_{\pm} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(\cos\theta\mp\frac{d}{2r}\sin^2\theta+\_-\frac{d}{2r}\cos^2\theta)(\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0))]

\end{equation}

\begin{equation}

V\_{\pm} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[\cos\theta\sin(

\omega t\_0) \pm \frac{\omega d}{2c}\cos^2\theta\cos(\omega t\_0) \pm \frac{d}{2r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

\newline\newline

We will call the sum of these two expressions \(V\_{T1}\).Adding together the two expressions we get

\begin{equation}

V\_{T1} = \frac{-p\_0\omega}{4\pi\epsilon\_0cr}[\frac{\omega d}{c}\cos^2\theta\cos(\omega t\_0)+\frac{d}{r}(\cos^2\theta-\sin^2\theta)\sin\omega t\_0)]

\end{equation}

\begin{equation}

V\_{T1} = \frac{-p\_0\omega^2 d}{4\pi\epsilon\_0c^2r}[\cos^2\theta\cos(\omega t\_0)+\frac{c}{\omega r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

And using our third approximation which said \(r >> \omega/c\) (equation 3), we can write \(V\_{T1}\) as

\begin{equation}

V\_{T1} = \frac{-p\_0\omega^2 d}{4\pi\epsilon\_0c^2r}[\cos^2\theta\cos(\omega t\_0)]

\end{equation}

and subbing back in \(t\_0 = t-\frac{r}{c}\) we get

\begin{equation}

V\_{T1} = \frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2r}\cos^2\theta\cos[\omega(t - \frac{r}{c})]

\end{equation}

Now we must look at the sum for \(V\_R\) and \(V\_L\). Similar to before, we will start by solving for the \(\sin[\omega(t-\frac{r}{c})]\) piece of our general equation (equation 30). Using our distances \(r\_r\) and \(r\_L\) we can write the following

\begin{equation}

\sin[\omega(t-\frac{r\_{RL}}{c})] = \sin[\omega(t-\frac{r}{c}(1\pm\frac{l}{2r}\sin\theta)]

\end{equation}

where

\begin{equation}

\sin[\omega(t-\frac{r}{c}(1\pm\frac{l}{2r}\cos\theta)] = \sin(\omega t\_0\mp\frac{\omega l}{2c}\sin\theta)

\end{equation}

which allows us to write

\begin{equation}

\sin[\omega(t-\frac{r\_{RL}}{c})] = \sin[\omega t\_0\pm\frac{\omega l}{2c}\sin\theta]

\end{equation}

where \(t\_0 = t -\frac{r}{c}\). Using the same trig identity as before (equation 50) we can write

\begin{equation}

\sin[\omega t\_0 \pm\frac{\omega l}{2c}\sin\theta]=\sin(\omega t\_0)\cos(\frac{\omega l}{2c}\sin\theta)\pm\cos(\omega t\_0)\sin(\frac{\omega l}{2c}\sin\theta)

\end{equation}

where

\begin{equation}

\cos(\frac{\omega l}{2c}\sin\theta) = 1

\end{equation}

and

\begin{equation}

\sin[\frac{\omega l}{2c}\sin\theta] = \frac{\omega l}{2c}\sin\theta

\end{equation}

Now we can finally write

\begin{equation}

\sin[\omega t\_0\pm\frac{\omega l}{2c}\sin\theta]=\sin(\omega t\_0)\pm\frac{\omega l}{2c}\sin\theta\cos(\omega t\_0)]

\end{equation}

From earlier, we solved for \(\cos\theta\_{RL}\). We can then plug in for the scalar potential which will yield the equation

\begin{equation}

V\_{RL} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(1\pm\frac{l}{2r}\sin\theta)(\sin\theta\mp\frac{l}{2r}\cos^2\theta)(\sin(\omega t\_0)\pm\frac{\omega l}{2c}\sin\theta\cos(\omega t\_0))

\end{equation}

\begin{equation}

V\_{RL} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(\sin\theta\mp \frac{l}{2r}\cos^2\theta\pm\frac{l}{2r}\sin^2\theta)(\sin(\omega t\_0)\pm\frac{\omega l}{2c}\sin\theta\cos(\omega t\_0))

\end{equation}

\begin{equation}

V\_{RL} = \frac{\mp p\_0\omega}{4\pi\epsilon\_0cr}[(\sin\theta

\sin(\omega t\_0)\pm\frac{\omega l}{2r}\cos^2\theta\sin(\omega t\_0)\pm\frac{l}{2r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

We will call the sum of this expression \(V\_{T2}\)

\begin{equation}

V\_{T2} = \frac{-p\_0\omega^2 l}{4\pi\epsilon\_0c^2r}[\frac{\omega l}{c}\cos^2\theta\sin(\omega t\_0)+\frac{l}{r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

and after factoring out \(\frac{\omega l}{c}\) we get

\begin{equation}

V\_{T2} = \frac{-p\_0\omega^2 l}{4\pi\epsilon\_0c^2r}[\cos^2\theta\sin(\omega t\_0)+\frac{c}{\omega r}(\cos^2\theta-\sin^2\theta)\sin(\omega t\_0)]

\end{equation}

And again using our third approximation which said \(r >> \omega/c\) (equation 3), we can write \(V\_{T2}\) as

\begin{equation}

V\_{T2} = \frac{-p\_0\omega^2 d}{4\pi\epsilon\_0c^2r}[\cos^2\theta\sin(\omega t\_0)]

\end{equation}

and subbing back in \(t\_0 = t-\frac{r}{c}\) we get

\begin{equation}

V\_{T2} = \frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2r}\cos^2\theta\sin[\omega(t - \frac{r}{c})]

\end{equation}

Now that we have the contribution of each dipole, we need to add them together to get the total scalar potential at point P.

\begin{equation}

V\_T = V\_++V\_-+V\_R+V\_L

\end{equation}

or

\begin{equation}

V\_T = V\_{T1} + V\_{T2}

\end{equation}

Subbing in for \(V\_{T1}\) and \(V\_{T2}\) we will get

\begin{equation}

V\_T = \frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2r}\cos^2\cos[\omega(t-\frac{r}{c})]+\frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2r}\cos^2\sin[\omega(t-\frac{r}{c})]

\end{equation}

after pulling out our common factors we can simplify the equation of our total scalar potential to be

\begin{equation}

V\_T\frac{-p\_0\omega^2d}{4\pi\epsilon\_0c^2r}\cos^2[d\cos[\omega(t-\frac{r}{c})]+l\sin[\omega(t-\frac{r}{c})]

\end{equation}

Now that we have finished solving for the scalar potential given by the dipoles, we must examine the vector potential given by each. The general form for the vector potential produced by an electric dipole can be seen below.

\begin{equation}

\vec{A} = \frac{-\mu\_0p\_0\omega}{4\pi r}\sin[\omega(t-\frac{r}{c})]]

\end{equation}

First, lets examine \(\vec{A}\_{+}\) and \(\vec{A}\_{-}\) for our dipoles on the y axis. We can write them as follows:

\begin{equation}

\vec{A\_{\pm}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r\_{\pm}}\sin[\omega(t-\frac{r\_{\pm}}{c})]

\end{equation}

\begin{equation}

\vec{A\_{\pm}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r}[(1\pm\frac{d}{2r}\cos\theta)[\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0)]]\hat{z}

\end{equation}

\begin{equation}

\vec{A\_{\pm}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r}[(\sin(\omega t\_0)\pm\frac{\omega d}{2c}\cos\theta\cos(\omega t\_0)\pm\frac{d}{2r}\cos\theta\sin(\omega t\_0)\hat{z}

\end{equation}

Now we can say \(\vec{A}\_{T1} = \vec{A}\_+ + \vec{A}\_-\).

\begin{equation}

\vec{A}\_{T1} = -\frac{\mu\_0p\_0\omega}{4\pi r}[\frac{\omega d}{c}\cos\theta\cos(\omega t\_0)+\frac{d}{r}\cos\theta\sin(\omega t\_0)]\hat{z}

\end{equation}

Then factoring out \(\frac{\omega d\cos\theta}{c}\) we are get

\begin{equation}

\vec{A}\_{T1} = - \frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta[\cos(\omega t\_0)+\frac{c}{\omega r}\sin(\omega t\_0)]]\hat{z}.

\end{equation}

And again in our radiation zone we can apply our third assumption (equation 3) which will eliminate the second term giving us

\begin{equation}

\vec{A}\_{T1} = -\frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta\cos(\omega t\_0)\hat{z}.

\end{equation}

and subbing back in for \(t\_0\)

\begin{equation}

\vec{A}\_{T1} = -\frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta\cos[\omega(t-\frac{r}{c})]\hat{z}.

\end{equation}

Next we will look at \(\vec{A}\_{R}\) and \(\vec{A}\_{L}\) for the dipoles on the x axis.

\begin{equation}

\vec{A\_{RL}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r\_{RL}}\sin[\omega(t-\frac{r\_{RL}}{c})]

\end{equation}

\begin{equation}

\vec{A\_{RL}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r}[(1\pm\frac{l}{2r}\sin\theta)(\sin(\omega t\_0)\pm\frac{\omega l}{2c}\sin\theta\cos(\omega t\_0))]\hat{z}

\end{equation}

\begin{equation}

\vec{A\_{RL}} = \mp \frac{\mu\_0p\_0\omega}{4\pi r}[(\sin(\omega t\_0)\pm\frac{\omega l}{2c}\sin\theta\cos(\omega t\_0)\pm\frac{l}{2r}\sin\theta\cos(\omega t\_0)]\hat{z}

\end{equation}

Similarly, we can now say

\begin{equation}

\vec{A}\_{T2} = \vec{A}\_R + \vec{A}\_L.

\end{equation}

\begin{equation}

\vec{A}\_{T2} = -\frac{\mu\_0p\_0\omega}{4\pi r}[\frac{\omega l}{c}\sin\theta\cos(\omega t\_0)+\frac{l}{r}\sin\theta\cos(\omega t\_0)]\hat{z}

\end{equation}

Then factoring out \(\frac{\omega l\sin\theta}{c}\) we are get

\begin{equation}

\vec{A}\_{T2} = -\frac{\mu\_0p\_0\omega^2l}{4\pi rc}\sin\theta[\cos(\omega t\_0)+\frac{c}{\omega r}\cos(\omega t\_0)]]\hat{z}.

\end{equation}

And applying our third assumption (equation 3) which will eliminate the second term giving us

\begin{equation}

\vec{A}\_{T2} = -\frac{\mu\_0p\_0\omega^2l}{4\pi rc}\sin\theta[\cos(\omega t\_0)]\hat{z}.

\end{equation}

and subbing back in for \(t\_0\)

\begin{equation}

\vec{A}\_{T2} = -\frac{\mu\_0p\_0\omega^2l}{4\pi rc}\sin\theta\cos[\omega(t-\frac{r}{c})]\hat{z}.

\end{equation}

And finally we can write

\begin{equation}

\vec{A}\_{Total} = \vec{A}\_{T1} + \vec{A}\_{T2}.

\end{equation}

\begin{equation}

\vec{A}\_{Total} = -\frac{\mu\_0p\_0\omega^2d}{4\pi rc}\cos\theta\cos[\omega(t-\frac{r}{c})]\hat{z} -\frac{\mu\_0p\_0\omega^2l}{4\pi rc}\sin\theta\cos[\omega(t-\frac{r}{c})]\hat{z}

\end{equation}

Factoring out the common terms we are left with

\begin{equation}

\vec{A}\_{Total} = -\frac{\mu\_0p\_0\omega^2}{4\pi rc}\cos[\omega(t-\frac{r}{c})](d\cos\theta+l\sin\theta)\hat{z}

\end{equation}

which is the final result for the vector potential given by the four dipoles. Now that we have solved for both the scalar and vector potentials we can plug them into our equations to find the electric and magnetic fields caused by this system of dipoles.

\subsection{Electric and Magnetic Fields}

Now that we have solved for the vector and scalar potentials at point P in the section above, we can use them to calculate the electric and magnetic fields produced by each dipole at point P.

The general equation for electric and magnetic fields can be seen below.

\begin{equation}

\vec{E}=-\vec{\bigtriangledown}V-\frac{\partial{\vec{A}}}{\partial{t}}

\end{equation}

\begin{equation}

\vec{B}=\vec{\bigtriangledown}\times\vec{A}

\end{equation}

First let us start off by solving for the gradient of the scalar potential

\begin{equation}

\vec{\bigtriangledown}V = \frac{\partial{V}}{\partial{r}}\hat{r}+\frac{1}{r}\frac{\partial{V}}{\partial{\theta}}\hat{\theta}

\end{equation}

\begin{equation}

\vec{\bigtriangledown}V =

\end{equation}

Next we will solve for the partial derivative of the vector potential

\begin{equation}

\frac{\partial{\vec{A}}}{\partial{t}} = \frac{\epsilon\_0p\_0\omega^3}{4\pi rc}\sin[\omega(t-\frac{r}{c}](d\cos\theta+l\sin\theta)\hat{z}

\end{equation}

which in spherical coordinates can be rewritten as

\begin{equation}

\frac{\partial{\vec{A}}}{\partial{t}} = \frac{\epsilon\_0p\_0\omega^3}{4\pi rc}\sin[\omega(t-\frac{r}{c}](d\cos\theta+l\sin\theta)(\cos\theta\hat{r}-\sin\theta\hat{\theta})

\end{equation}

Putting these pieces together we can solve for the electric field

\begin{equation}

\vec{E} =

\end{equation}

Now to find the magnetic field

\begin{equation}

\vec{B} = \vec{\bigtriangledown}\times\vec{A} = \frac{1}{r}[\frac{\partial{\vec{A\_{\theta}}}}{\partial{t}}-\frac{\partial{\vec{A\_{r}}}}{\partial{\theta}}]\hat{\phi}

\end{equation}

\begin{equation}

\vec{B} =

\end{equation}

\subsection{Poynting Vector}

Next lets look at the energy radiated by the electric dipoles at point P. To do this, we will need to find the Poynting vector which represents the directional energy flux. The general formula for the Poynting Vector can be seen below.

\begin{equation}

\vec{S}=\frac{1}{\mu\_0}(\vec{E}\times\vec{B})

\end{equation}

\subsection{Radiated Power}

Now lets examine the total radiated power by the system at point P. We will do this by integrating the average intensity using the Poynting vector which was found in Section 3.3.

\begin{equation}

P=\int\_{a}^{b}<\vec{S}>\cdot d\vec{a}

\end{equation}

In order to find \(<\vec{S}>\) we will need to average the Poynting vector over one cycle.

\subsection{Comparing Intensity Profiles}

[Show all 3 radiation plots in one image here]

\section{Conclusion}

(go over results of paper)\newline

(relate to engineering topics here, such as antennas or phased array. Emphasize importance)

\section{Appendix}

\subsection{Maple Code}

(put maple code here used for symbolic computation)

\subsection{MATLAB Code}

(put MATLAB code here used for graphing and plotting)

\section{References}

Griffiths, D. J. (1999) Introduction to Electrodynamics Third Edition. Upper Saddle River, New Jersey: Prentice-Hall.

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